Discovering Math: Number Theory
Teacher's Guide

Grade Level: 6–8	**Curriculum Focus:** Mathematics	**Lesson Duration:** 3–4 class periods

Program Description

Discovering Math: Number Theory — From equivalent number representations to rational numbers and their subsets to exponents and scientific notation, introduce students to more advanced properties and concepts of numbers and divisibility rules.

Lesson Plan

Student Objectives

- Classify numbers as whole, rational, irrational, positive, or negative and identify examples of each.
- Classify numbers as prime or composite and recognize relatively prime integers.
- Create a representation to demonstrate an understanding of the relationships between numbers.
- Perform factorization.
- Test numbers for divisibility by common factors.
- Convert numbers to and from scientific notation.
- Demonstrate an understanding of the advantages of scientific notation.
- Demonstrate and understanding of the binary number system and convert numbers to and from bases two and ten.

Materials

- *Discovering Math: Number Theory* video
- Newspapers, magazines, brochures, etc.
- Posterboard
- Calculators
- State Population Chart (see below)
- Converting Numbers Chart (see below)

Procedures

1. Display the following terms:

   whole number

   rational number

   irrational number

   positive number

   negative number

   Ask students to explain and give examples of each type of number.

   - Tell students they will create a poster to display each type of number. Provide magazines, newspapers, advertisements, and other print materials for students to look through and find at least ten examples of different types of numbers. Make sure each type of number is represented at least once. Have the students cut out the examples and display and label them on their poster.
   - When students have completed their posters, have students share and explain their work. The posters can be displayed in the classroom for reference after the lesson.

2. Ask students what they know about a number if it is prime. Can they tell if it is rational, irrational, or even? Have them share and explain their ideas. Remind them of information presented in the video regarding relationships among numbers.

   - Have students write at least three relationships of the form "If a number is ___, it must [not] be ____."
   - Ask them to recall the representation from the video that explained the relationship between numbers.
   - Have students create their own representation to display and explain the relationship statements they wrote.
   - Have them share their representations with the class.

3. Review divisibility rules with students. Ask them to describe the rules they can use to determine if a number is divisible by three, five, or nine.

   Tell students they will create divisibility challenges for each other.

   - Have them use calculators to multiply large numbers by three, five, or nine in order to generate large multiples of three, five, and nine. Have each student make a list of at least ten numbers.
   - Assign each student a partner. Have them switch lists and use divisibility rules to figure out which numbers on the list are divisible by three, five, or nine.

- Remind students that divisibility rules work both ways. For example, if a number is divisible by three, its digits sum to a multiple of three. And if a number's digits sum to a multiple of three than it is a multiple of three.

  Pose this question:

  > Julie is walking to a friend's house when it starts to rain, smearing the paper on which she had written the address, 85**?**4 Main Street. The third digit of the house number is now illegible. If she remembers that the house number was a multiple of nine, can she figure out the number? If so, how?

  Ask students work with their partner to solve the problem and then present their solution and explanation to the class.

4. Display the number 24,000,000 and ask students to read the number. Do they know another way to represent the number? Elicit responses and discuss the use of scientific notation.
   - Model how to write 24,000,000 in scientific notation (2.4×10^7). Ask students to convert the following numbers into scientific notation:

     378,000

     1,5000,000,000

     56,000,000

     134,000,000,000

     42,000,000

     1,000
   - Distribute copies of the State Population Chart. Have students choose 15 states. Provide a resource they can use to find the population of each state. Have them complete the chart by filling in the population and then writing the number in scientific notation.
5. Ask students to recall the binary (or base 2) system presented in the video. Discuss how to convert a number from base 10 to the binary number system.
   - Model the example:

     Begin by setting up and explaining the base 2 place value chart.

     Convert 1011 (base 2) to base 10

     1010 base 2 = 10 base 11

     Convert 14 (base 10) to base 2

     14 base 10 = 1110 base 2
   - Model more examples until students are comfortable with the procedure.
   - Have them complete the Converting Numbers Chart.

Assessment

Use the following three-point rubric to evaluate students' work during this lesson.

- **3 points:** Students were highly engaged in class discussions; clearly demonstrated the ability to identify and describe a variety of types of numbers; clearly demonstrated a working knowledge of scientific notation and the binary system; and clearly demonstrated an understanding of the relationship that exists among numbers.
- **2 points:** Students participated in class discussions; satisfactorily demonstrated the ability to identify and describe a variety of types of numbers at least 80% of the time; satisfactorily demonstrated a working knowledge of scientific notation and the binary system and were able to use this knowledge correctly at least 80% of the time; and satisfactorily demonstrated an understanding of the relationship that exists among numbers.
- **1 point:** Students participated minimally in class discussions; demonstrated the ability to identify and describe a variety of types of numbers less than 80% of the time; did not demonstrate a working knowledge of scientific notation and the binary system; and was unable to demonstrate an understanding of the relationship that exists among numbers.

Vocabulary

binary system

Definition: the base 2 number system in which place value increases by powers of 2 instead of 10
Context: Sam converted 19 to the binary system, 10011.

composite number

Definition: a number having at least one factor other than itself and 1; a number that is not prime
Context: Four, six , and eight are composite numbers.

integer

Definition: natural numbers used for counting
Context: Numbers on a thermometer are integers.

prime

Definition: a number having no factors other than itself and 1
Context: Sue concluded that 17 is a prime number because it has no factors other than 1 and 17.

rational number

Definition: a number that can be expressed as a whole number or as the quotient of two whole numbers
Context: Zoë determined that 0.375 is a rational number because it can also be expressed as $\frac{3}{8}$.

relatively prime

Definition: having no common factors other than 1

Context: Manuel has 50 pencils and 49 markers. He would be able to distribute the pencils evenly to a class of 2, 5, 10, 25, or 50 students, and he would be able to distribute the markers evenly to a class of 7 or 49 students. He would never be able to distribute both the pencils and markers evenly because 50 and 49 are relatively prime.

scientific notation

Definition: a system of writing numbers as the product of a number between 1 and 10 and a power of 10

Context: Ben displayed a measurement in scientific notation as 8.49×10^{-4}.

Academic Standards

Mid-continent Research for Education and Learning (McREL)
McREL's Content Knowledge: A Compendium of Standards and Benchmarks for K–12 Education addresses 14 content areas. To view the standards and benchmarks, visit http://www.mcrel.org/compendium/browse.asp.

This lesson plan addresses the following benchmarks:

- Understands the relationships among equivalent number representations (e.g., whole numbers, positive and negative integers, fractions, ratios, decimals, percents, scientific notation, exponentials) and the advantages and disadvantages of each type of representation.
- Understands the characteristics and properties (e.g., order relations, relative magnitude, base-ten place values) of the set of rational numbers and its subsets (e.g., whole numbers, fractions, decimals, integers).
- Understands the role of positive and negative integers in the number system.
- Uses number theory concepts (e.g., divisibility and remainders, factors, multiples, prime, relatively prime) to solve problems.
- Understands the characteristics and uses of exponents and scientific notation.
- Understands the structure of numeration systems that are based on numbers other than 10 (e.g., base 60 for telling time and measuring angles, Roman numerals for dates and clock faces).
- Understands the concepts of ratio, proportion, and percent and the relationships among them.

National Council of Teachers of Mathematics (NCTM)
The National Council of Teachers of Mathematics (NCTM) has developed national standards to provide guidelines for teaching mathematics. To view the standards online, go to http://standards.nctm.org.

This lesson plan addresses the following standards:

- Understand and use ratios and proportions to represent quantitative relationships.
- Develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation.
- Develop meaning for integers and represent and compare quantities with them.
- Select, apply, and translate among mathematical representations to solve problems.
- Use factors, multiples, prime factorization, and relatively prime numbers to solve problems.

Support Materials

Develop custom worksheets, educational puzzles, online quizzes, and more with the free teaching tools offered on the Discoveryschool.com Web site. Create and print support materials, or save them to a Custom Classroom account for future use. To learn more, visit

- http://school.discovery.com/teachingtools/teachingtools.html

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DVD Content

This program is available in an interactive DVD format. The following information and activities are specific to the DVD version.

How to Use the DVD

The DVD starting screen has the following options:

***Play Video*—**This plays the video from start to finish. There are no programmed stops, except by using a remote control. With a computer, depending on the particular software player, a pause button is included with the other video controls.

***Video Index*—**Here the video is divided into chapters indicated by title. Each chapter is then divided into four sections indicated by video thumbnail icons; brief descriptions are noted for each section. To play a particular segment, press Enter on the remote for TV playback; on a computer, click once to highlight a thumbnail and read the accompanying text description and click again to start the video.

***Quiz*—**Each chapter has four interactive quiz questions correlated to each of the chapter's four sections.

***Standards Link*—**Selecting this option displays a single screen that lists the national academic standards the video addresses.

***Teacher Resources*—**This screen gives the technical support number and Web site address.

Video Index

I. Number Representations (9 min.)

Number Representations: Introduction
Explore the ways to represent a number — using fractions, decimals, percents, or scientific notation.

Example 1: Integers
Integers are natural numbers used for counting. See how the numbers on a thermometer are integers, as well as how temperature is reported using integers.

Example 2: Scientific Notation
Watch as scientific notation is used to express, compute, and compare very large and small numbers.

Example 3: Fractions, Decimals, and Percents
One number can be expressed equivalently as a fraction, decimal, or percent. Learn how the type of number used depends on the situation.

II. Rational Numbers (9 min.)

Rational Numbers: Introduction

Examine rational numbers and see how they can be written as $\frac{a}{b}$ where a and b are integers and b does not equal zero.

Example 1: Whole Numbers as Rational Numbers
See how rational numbers can be expressed as fractions as long as the denominator is not zero and learn why positive and negative integers are rational numbers.

Example 2: Fractions
Explore how fractions represent parts of a whole or a group. See why a fraction is a rational number and zero can never be the denominator.

Example 3: Decimals, Terminating and Repeating
See why fractions expressed as decimals will either be terminating or repeating. A terminating decimal has a final digit. A repeating decimal has one or more digits that repeat without end.

III. Role of Integers (9 min.)

Role of Integers: Introduction
Investigate the properties of integers, all whole numbers and their opposites. See how a number line represents integers and their distance from zero.

Example 1: Positive Integers
Take a closer look at positive integers — whole numbers greater than zero that can be used to count things.

Example 2: Negative Integers
Take a closer look at negative integers — numbers less than zero. See how subtracting an integer is the same as adding its opposite.

Example 3: Roles for Integers
See how integers are used to express factors and multiples and explore the relationship between natural numbers, whole numbers, integers, and rational numbers.

IV. Solving Problems Using Number Theory (8 min.)

Solving Problems Using Number Theory: Introduction
Explore the parts of a mixed number. See how an improper fraction with a denominator that is not a factor of the numerator can be expressed as a mixed number.

Example 1: Factors and Multiples
See how factors and multiples are used to determine the number and cost of tickets needed for rides at the fair.

Example 2: Divisibility and Remainders
Investigate how a number is divisible by a divisor if there is no remainder in the quotient.

Example 3: Prime and Relatively Prime Numbers
Take a look at prime numbers and see why two numbers are relatively prime if their only common factor is one and their least common multiple is their product.

V. Exponents and Scientific Notation (8 min.)

Exponents and Scientific Notation: Introduction
Investigate how scientific notation is used to express very large numbers.

Example 1: Exponents
Use exponents to express the number of times a base number is used in a multiplication.

Example 2: Scientific Notation
Use scientific notation to express, compute, and compare very large and small numbers.

Example 3: Scientific Notation for Very Low Numbers
See how scientific notation is used to express very small numbers, such as the mass of a DNA molecule.

VI. Numeration Systems (11 min.)

Numeration Systems: Introduction
Examine how the symbols used to represent numbers are used to create a numeration system.

Example 1: Roman Numerals
Explore the history of Roman numerals and see how the Roman system uses addition and subtraction by the relative position of the symbols to express a value.

Example 2: Base 20 and Base 60
Take a look at the Mayan base 20 system and the Babylonian base 60 system that is used today to measure time.

Example 3: Binary Number Systems
Investigate the binary system — a base 2 system where 1 is the greatest value in any place. Then convert numbers from the binary system to the base 10 system.

VII. Ratio, Proportion, and Percent (9 min.)

Ratio, Proportion, and Percent: Introduction
Explore how ratios express the relationship of one amount to another.

Example 1: Ratio
Take a closer look at ratios and see how they can be expressed using a colon (3:5).

Example 2: Percents
See how a fraction can be used to express a ratio and convert a fraction to a percent.

Example 3: Proportions
A proportion is a statement that two ratios are equal and can be expressed with a colon or an equal sign.

Quiz

I. Number Representations

1. Identify the integer.
   A. $\frac{1}{2}$
   B. -5.4
   C. $6\frac{3}{4}$
   D. -9

   Answer: D

2. Identify the correct way to express 3,450,000 in scientific notation.
   A. 3.45×10^6
   B. 345×10^6
   C. 3.45×10^4
   D. 3.45×10^5

   Answer: A

3. Evaluate the expression and choose the correct answer.
   $\frac{1}{4} + \frac{1}{2} =$ ____
   A. 25%
   B. $\frac{2}{6}$
   C. 3.4
   D. 75%

   Answer: D

II. Rational Numbers

1. Which of the following is *not* a rational number?
   A. $\frac{8}{0}$
   B. $-\frac{3}{4}$
   C. 0.68
   D. 236

   Answer: A

2. Identify another way to represent the number 8.
   A. $\frac{4 \times 3}{2 \times 2}$ C. $\frac{4 \times 8}{2 \times 4}$
   B. $\frac{8}{0}$ D. $\frac{1}{8}$

   Answer: C

3. There are seven red, five blue, nine green, and three yellow marbles in a bag. What fraction of the marbles are either red or green?
   A. $\frac{7}{24}$ C. $\frac{3}{8}$
   B. $\frac{1}{3}$ D. $\frac{2}{3}$

   Answer: D

4. Frank wants to express $\frac{1}{3}$ as a decimal and has calculated 0.3333333 so far. What should he know?
   A. $\frac{1}{3}$ converts to a terminating decimal with no final digit
   B. $\frac{1}{3}$ converts to a repeating decimal with no final digit
   C. $\frac{1}{3}$ converts to a repeating decimal with a final digit
   D. $\frac{1}{3}$ can't be expressed as a decimal

   Answer: B

III. Role of Integers

1. What is the absolute value of -5?
   A. -5
   B. 0
   C. 5
   D. 5^2

   Answer: C

2. Paul wants to place only positive integers on his number line. Which of the following numbers could he use?
   A. $-53\frac{1}{2}$
   B. 0
   C. 27.5
   D. 49

   Answer: D

3. Which statement is true?
   A. All integers are whole numbers.
   B. All natural numbers are whole numbers.
   C. All rational numbers are whole numbers.
   D. All rational numbers are natural numbers.

   Answer: B

IV. Solving Problems Using Number Theory

1. Which fraction can also be expressed as a mixed number?
   A. $\frac{16}{4}$ C. $\frac{32}{7}$
   B. $\frac{25}{5}$ D. $\frac{45}{9}$

   Answer: C

2. Jason, Sue, and Hunter are at the fair. They would like to ride five rides and each ride costs four tickets. How many tickets do they need in all?
   A. 20
   B. 60
   C. 80
   D. 120

   Answer: B

3. Identify the remainder.
   $67 \div 8 =$ ___
   A. 8
   B. 7
   C. 3
   D. 1

   Answer: C

4. Which are relatively prime numbers?
   A. 6 and 23
   B. 6 and 12
   C. 4 and 18
   D. 7 and 49

   Answer: A

V. Exponents and Scientific Notation

1. George is thinking of a number and has expressed it using scientific notation. 7×10^6 What number is he thinking of?
   A. 76
   B. 700
   C. 700,000
   D. 7,000,000

   Answer: D

2. $4.2^3 =$ ____
   A. 12.6
   B. 17.64
   C. 74.088
   D. 126

   Answer: C

3. Identify the correct way to express 0.000007 in scientific notation.
   A. 7×10^{-6}
   B. 7×10^{6}
   C. 0.7×10^{6}
   D. 0.7×10^{-6}

   Answer: A

VI. Numeration Systems

1. What numeration system do students in the United States use?
   A. digit system
   B. Arabic system
   C. Roman system
   D. numeral system

   Answer: B

2. Chris sees the Roman numeral CDVI. What is the value of this Roman numeral?
   A. 406
   B. 456
   C. 604
   D. 651

   Answer: A

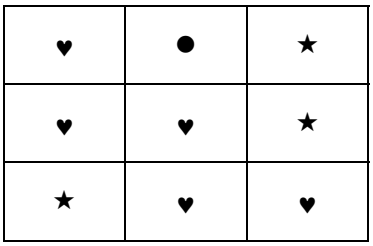
3. Convert the following base 2 number into a base 10 number.
   10101
   A. 20
   B. 21
   C. 26
   D. 41

   Answer: B

VII. Ratio, Proportion, and Percent

1. What is the ratio of stars to hearts?
   A. 1:3
   B. 5:3
   C. 8:9
   D. 3:5

   Answer: D



2. Oliver drinks 12 cups of liquid a day. Nine of those are water and the rest are milk or juice. What percent of Oliver's daily liquid intake is water?
   A. 9%
   B. 25%
   C. 75%
   D. 85%

   Answer: C

3. The ratio of teachers to students at Old Mill Middle School is 2:16. If there are 80 students, how many teachers are needed to maintain this ratio?
   A. 5
   B. 7
   C. 8
   D. 10

   Answer: D

State Population Chart

State	Population	Population in Scientific Notation

Converting Numbers Chart

	Base 2 Number	Base 10 Number
1	1010	
2	1110001	
3	100001	
4	1010111	
5	1001	
6	10001	
7	1010101	
8	100010001	
9	1001001	
10	1101101	
11	11100111	
12	10011001	
13	11011	
14	11000	
15	1000001	
16	1010111	
17		19
18		16
19		52
20		24
21		35
22		42
23		5
24		26
25		14
26		23
27		9
28		72
29		83
30		60
31		40
32		28

Answer Key — Converting Numbers Chart

	Base 2 Number	Base 10 Number
1	1010	10
2	1110001	113
3	100001	33
4	1010111	87
5	1001	9
6	10001	17
7	1010101	85
8	100010001	273
9	1001001	73
10	1101101	109
11	11100111	231
12	10011001	281
13	11011	27
14	11000	24
15	1000001	65
16	1010111	87
17	10011	19
18	10000	16
19	110100	52
20	11000	24
21	100011	35
22	101010	42
23	101	5
24	11010	26
25	1110	14
26	10111	23
27	1001	9
28	1001000	72
29	1010011	83
30	111100	60
31	101000	40
32	11100	28